ISOTHERMAL CREEP OF A BINARY GAS MIXTURE ALONG A FLAT SURFACE

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The method of half-space moments [1] is used to derive an analytic expression for the velocity of isothermal creep of a binary gas mixture along a flat surface. The distribution functions for the gas molecules are found from the solution of the Boltzmann kinetic equation with a model collision integral.

We assume that a binary mixture of gases with densities n_1 and n_2 and molecular masses m_1 and m_2 fills the x > 0 half-space above the x = 0 plane. Far from the surface, the gas mixture is moving along the y axis at some average-mass velocity $u_\infty = u_0 + (du/dx)_\infty x$, where the gradient of the average-mass velocity, $(du/dx)_\infty$, is assumed given, and u_0 is the velocity of isothermal creep, which is to be determined. We also assume

$$u_\infty \ll \bar{v}_i. \tag{1}$$

Here $\bar{v}_i$ is the average thermal velocity of component i in the mixture. The medium is assumed homogeneous along the z direction.

The behavior of the system is governed completely by the distribution functions f_i for the molecules of component i. These functions are the solutions of a system of two Boltzmann kinetic equations, which can be written in this case as [2]

$$v_{xi}\frac{\partial f_i}{\partial x} + v_{yi}\frac{\partial f_i}{\partial y} = \nu_{ii}(M_i - f_i) + \nu_{ij}(\tilde{M}_i - f_i) \quad (i = 1,\ 2). \tag{2}$$

Here

$$\tilde{M}_i = n_i\left(\frac{m_i}{2\pi kT}\right)^{3/2}\exp\left\{-\frac{m_i[v_{xi}^2 + (v_{yi} - \tilde{u}_i)^2 + v_{zi}^2]}{2kT}\right\};$$

$$M_i = n_i\left(\frac{m_i}{2\pi kT}\right)^{3/2}\exp\left\{-\frac{m_i[v_{xi}^2 + (v_{yi} - u_i)^2 + v_{zi}^2]}{2kT}\right\};$$

$$\tilde{u}_i = \mu_1 u_1 + \mu_2 u_2; \quad \mu_i = \frac{m_i}{m_1 + m_2} \quad (i = 1,\ 2);$$

ν_{ii} and ν_{ij} are the analogs of the frequencies of collisions between molecules of the same species and between different molecules, and $\vec{v}_i$ is the velocity of the molecules of component i.

The quantities n_i, u_i, and T are determined from

$$\rho_i = m_i n_i = m_i \int f_i(x,\ y,\ \vec{v})\, d\vec{v}_i,$$

$$u = \frac{\rho_1 u_1 + \rho_2 u_2}{\rho_1 + \rho_2} = \frac{1}{\rho_1 + \rho_2}\left(m_1 \int f_1 v_{y1} d\vec{v}_1 + m_2 \int f_2 v_{y2} d\vec{v}_2\right),$$

$$T = \frac{1}{3(n_1 + n_2)}\left[m_1 \int (\vec{v}_1 - \vec{u})^2 f_1 d\vec{v}_1 + m_2 \int (\vec{v}_2 - \vec{u})^2 f_2 d\vec{v}_2\right].$$

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and integrating over the entire corresponding velocity space, we find a system of differential equations for the functions $a_{0i}^{\pm}(x)$ and $a_{1i}^{\pm}(x)$:

$$\pm\frac{1}{4}\frac{da_{0i}^{\pm}}{dx}+\frac{1}{8}\sqrt{\pi}\frac{da_{1i}^{\pm}}{dx}=K_i\left[\frac{1}{8}\sqrt{\pi}\,(a_{0i}^{+}+a_{0i}^{-})+\frac{1}{8}(a_{0i}^{+}-a_{0i}^{-})-\frac{1}{4}\sqrt{\pi}\,a_{0i}^{\pm}\mp\frac{1}{4}a_{1i}^{\pm}\right]\quad(i=1,\ 2),$$

$$\frac{1}{8}\sqrt{\pi}\frac{d}{dx}a_{0i}^{\pm}\pm\frac{d}{dx}a_{1i}^{\pm}=K_i\left[\pm\frac{1}{8}(a_{0i}^{+}+a_{0i}^{-})\pm\frac{1}{8\sqrt{\pi}}(a_{1i}^{+}-a_{1i}^{-})\mp\frac{1}{4}a_{0i}^{\pm}-\frac{1}{8}\sqrt{\pi}\,a_{1i}^{\pm}\right].\tag{8}$$

Here the parameter K_i is determined from the boundary condition at infinity, i.e., from the requirement that the functions in (4) satisfy system (2) in the limit $x\to\infty$, where $\Phi_i\to 0$. From this condition we find

$$K_i=\left(\frac{m_i}{2kT}\right)^{1/2}(\nu_{ii}+\nu_{ij})=\frac{2}{B_i}\quad(i=1,\ 2).\tag{9}$$

The solution of Eq. (8) is

$$a_{0i}^{+}=C_ie^{-\alpha_i x},\quad a_{0i}^{-}=\alpha_{1i}e^{-\alpha_i x},\quad a_{1i}^{+}=\alpha_{2i}e^{-\alpha_i x},\quad a_{1i}^{-}=\alpha_{3i}e^{-\alpha_i x},$$

where

$$\alpha_i=\sqrt{2}\,\frac{\pi-2}{4-\pi}\,K_i;$$

$$\alpha_{1i}=C_i-\frac{\sqrt{\pi}\,\alpha_i}{K_i(\pi-2)}\left[\frac{4\sqrt{\pi}\,\alpha_1+4K_i-2K_i\pi-\pi^{3/2}\alpha_i}{\sqrt{\pi}\,\alpha_i+2K_i-K_i\pi}\right]C_i=\beta_{1i}C_i;$$

$$\alpha_{2i}=-\frac{(\pi-2)\,\alpha_i}{\sqrt{\pi}\,\alpha_i+2K_i-K_i\pi}\,C_i=\beta_{2i}C_i;$$

$$\alpha_{3i}=\left\{\frac{(\pi-2)\,\alpha_i}{\sqrt{\pi}\,\alpha_i+2K_i-K_i\pi}-\frac{2\alpha_i}{K_i(\pi-2)}\left[\frac{4\sqrt{\pi}\,\alpha_i+4K_i-2K_i\pi-\pi^{3/2}\alpha_i}{\sqrt{\pi}\,\alpha_i+2K_i-K_i\pi}\right]\right\}C_i=\beta_{3i}C_i.$$

The constants C_1 and C_2 and the quantities u_{10} and u_{20} are found from boundary conditions (3) to be

$$C_i=\frac{2-q_i}{\beta_{2i}+(1-q_i)\,\beta_{3i}}\,B_i\left(\frac{du}{dx}\right)_{\infty},\tag{10}$$

$$u_{i0}=\frac{(1-q_i)\,\beta_{1i}-1}{2q_i}\left[\frac{2-q_i}{\beta_{2i}+(1-q_i)\,\beta_{3i}}\right]B_i\left(\frac{du}{dx}\right)_{\infty}\left(\frac{2kT}{m_i}\right)^{1/2}.\tag{11}$$

Using (11), we find the creep velocity of a binary gas mixture along a flat surface to be

$$u_0=\frac{\sqrt{2kT}}{\rho}\left\{\frac{(1-q_1)\,\beta_{11}-1}{2q_1}\left[\frac{2-q_1}{\beta_{21}+(1-q_1)\,\beta_{31}}\right]n_1\sqrt{m_1}\,B_1+\frac{(1-q_2)\,\beta_{21}-1}{2q_2}\left[\frac{2-q_2}{\beta_{22}+(1-q_2)\,\beta_{32}}\,n_2\sqrt{m_2}\,B_2\right]\right\}\left(\frac{du}{dx}\right)_{\infty}.\tag{12}$$

We turn now to the case of the isothermal creep of a single-component gas. For this case we let the density of one component of the mixture (say, the second) vanish. Then the second term in (12) vanishes, B_1 and ρ become

$$B_1=\frac{5}{4}\sqrt{\pi}\,\lambda,\quad \rho=n_1m_1.\tag{13}$$

Then from (12) we find

$$u_0=\frac{5}{4}\sqrt{\pi}\,\lambda\left[\frac{(1-q_1)\,\beta_{11}-1}{2q_1}\right]\frac{2-q_1}{\beta_{21}+(1-q_1)\,\beta_{31}}\left(\frac{du}{dx}\right)_{\infty},\tag{14}$$

where

$$\beta_{11}=0.230;\quad \beta_{21}=-0.979;\quad \beta_{31}=0.110.\tag{15}$$

In the case q = 1 (the case of purely diffuse reflection), we find from (14), using (15),

$$u_0 = 1.106\lambda \left(\frac{du}{dx}\right)_\infty . \tag{16}$$

This result differs by only 2% from the isothermal creep velocity found in [4].

LITERATURE CITED

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